Consumer rationality assumptions in the real-time pricing of electricity

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Abstract: Dynamic tariffs such as RTP (real-time pricing) and day-ahead pricing function as load management tools because they interact with consumer behaviour. Hence analytical models for an electricity supply system with RTP would need to incorporate behavioural models for consumers. These models have to be logically and mathematically consistent and empirically meaningful. The model developed here relies on the concepts of demand elasticity across time, degree of consumer economic rationality and, on the supply side, on the price formation model. The paper explores a range of assumptions in respect of these matters.

1 Introduction

There are several reasons suggested in favour of real-time pricing [1] where the price signal is set to reflect the instantaneous cost of production. Central among these are the following:

(i) As a load management tool, this type of pricing redistributes demand away from expensive production periods to other times when more cost-effective generators can pick up some demand. Hence the same amount of total (annual, daily, etc) energy can be produced at less cost.

(ii) As a capital conservation tool, this type of tariff helps to reduce annual peak demand for the same annual gross energy production. This results in lower average overall kWh costs.

(iii) As a consumption rationalisation tool, base period consumers are relieved of the burden of subsidising peak period consumers, which occurs with present day average production cost based tariffs. In general, rational consumers will, at any time, pay more/less and consume less/more, depending on how important consumption at any particular time of day/week/season is to them.

These basic considerations spin-off to several other effects. These include:

(*a*) Mediating the revenue collection side of a future competitive electricity supply system [2],

(*b*) Assisting rational operation of off-peak energy storage, pumped storage plant scheduling and inter-area energy exchange, and

(*c*) Assisting system control and recovery during periods of emergency or shortage.

This paper focuses on one specific aspect of real-time pricing which is closely tied in with (iii) above. This concerns different possible types of consumer behaviour (price response) models and their effect on the overall supplier-consumer price-demand interaction. The paper is based on mathematical simulations which make a variety of different assumptions about possible consumer behaviour and then proceed to investigate these cases. At the present time it is not possible to make reliable statements about the validity of the different assumptions as statistical information about consumer behaviour can be accumulated only when real-time tariffs are put into operation. Furthermore, as consumer behaviour will develop over time as familiarity with this type of tariff (and software and hardware supportive of flexible response) grows, the studies presented may be viewed as possible different scenarios and stages of a dynamic and evolving situation [3, 4].

2 Consumer behaviour models

In general the price sensitivity of consumer demand is represented by the notion of price elasticity of demand, $e = dD/dp$, where D is the per unit demand and p is the per unit price — some values of demand and price being selected as the base units. The simple economic concept of price elasticity of demand simply says that demand for goods falls if the price rises, and *vice versa*. However, with real-time pricing of electricity, this concept has to be extended [5, 6] to take account of the fact that we are now dealing with a time-dependent phenomenon. Consumption at any one time depends not only on the price at that time but also on the price at other times. For example, a domestic consumer may defer some activity for a period if a possible saving is perceived, or an industry may be able to reschedule production to the extent that this is feasible. The essence of real-time pricing as a load management tool is the expectation that consumers will shift consumption from one time period to another if there is a commensurate price attraction for doing so.

Hence, the elasticity concept is extended as follows. The total time period under scrutiny, such as a day or a week, is divided into several time-steps (say hourly)

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corresponding to the time intervals at which the tariff is updated. If t, t' denote two such steps and if T stands for the whole duration, let

$$e_{tt'} = \frac{\partial D_t}{\partial p_{t'}} \quad \forall t, t' \tag{1}$$

As current demand changes inversely with current price it is natural to expect that $e_{tt} < 0$. As price reductions (increases) at other times will cause current demand to fall (rise) it is expected that $e_{tt'} \geqslant 0$ if $t \neq t'$. If at time t consumers are indifferent to, or have no knowledge of, some future time t', $e_{tt'} = 0$.

It is to be observed that the partial derivative notation has now been introduced. This is because D_t, the demand at time t, is no longer a function of p_t only, but is also a function of $p_{t'}$ for all t' in T. Hence e_{tt} may be referred to as self-elasticity and $e_{tt'}$ $t \neq t'$ as the cross-time elasticity of demand.

Secondly, we assume that these coefficients remain constant for the purposes of any one study. In this case we can form a $T \times T$ elasticity matrix

$$\boldsymbol{E} = [e_{tt'}] \tag{2}$$

and if $\boldsymbol{E}$ is non-singular, define the inverse matrix $\boldsymbol{F}$ by

$$\boldsymbol{F} = [f_{tt'}] = \boldsymbol{E}^{-1} \tag{3a}$$

where, obviously

$$f_{tt'} = \frac{\partial p_t}{\partial D_{t'}} \quad \forall t, t' \tag{3b}$$

It will be found that numerically, almost always, all coefficients $f_{tt'}$ are negative. Mathematically this is likely to be otherwise only if $|e_{tt}| < e_{tt'}(t \neq t')$ or if $e_{tt'} < 0(t \neq t')$ for some coefficients at least. However, neither of these cases is of practical significance.

A further issue that needs to be considered is the range of price and demand variation for which it is assumed that these coefficients remain constant. Clearly, if the demand at any particular time varies far from its normal operating range, it is likely to become less and less sensitive to price variations at other times. Hence it is assumed that $e_{tt'}(t \neq t')$ remains constant for demand variations $\pm\eta D_t$ around the normal operating range of D_t. The coefficient η is such that $0 \leqslant \eta < 1$ and usually taken to be small. Values of η in the range 0.05 to 0.2 were considered in this study. Outside this range, that is far from normal operating conditions, consumers are assumed to have become insensitive to price variation; that is, mathematically, $e_{tt'} = 0$ outside this range for $t \neq t'$. No particular assumptions about the range within which self-elasticity (e_{tt}) remains constant is mathematically necessary.

A final point in the representation of consumer behaviour pertains to the time horizon of consumer rationality. The perfect consumer is defined to be the one who takes a long range (***LR***) view in decision making. That is, the ***LR*** consumer decides how much to pay and what quantity to consume at each time-step so as to maximise overall (long-term) considerations. The other extreme is the very short-sighted or spot-rational (***SR***) consumer who sets consumption and willingness to pay from considerations pertaining to the current time-step only.

Conventional supply-demand economics is based on the notion of consumer utility. Utility may be defined as the value (expressed in monetary units) that a consumer derives from the use of a good. Hence if D is the amount consumed, the utility is expressed as a function $B(D)$. Now B will increase with D but at a declining rate; the 11th pair of shoes is a good deal less important than the first. Defining marginal utility as dB/dD, the following crucial behavioural description of 'the rational consumer' is introduced. 'The price that a rational consumer is willing to pay (p) = perceived consumer marginal utility (dB/dD) of the last unit purchased'.

Hence, conventional economics asserts that supply-demand equilibrium will prevail under perfect market conditions when the unit price (p) that sellers need to charge and the marginal utility (dB/dD) that consumers derive from the last unit consumed, are equal to each other.

Now using the symbol B_τ to represent consumer utility at time-step τ, the two types of consumers described above may, mathematically, be represented as follows

LR *consumer:*

$$p_t = \frac{\partial}{\partial D_t}\left(\sum_{\tau \in \{T\}} B_\tau\right) \tag{4}$$

SR *consumer:*

$$p_t = \frac{\partial}{\partial D_t}(B_t) \tag{5}$$

Observe that eqn. 4 says that at time t, current price is determined by the marginal global-utility with respect to current consumption, while eqn. 5 says that price at time t depends only on marginal current utility with respect to current consumption. In eqn. 4, $\{\boldsymbol{T}\}$ stands for the entire time domain of interest.

3 Real-world consumers

3.1 Consumer perception of rationality

The concepts of a perfect (***LR***) and at the other extreme a spot-rational (***SR***) consumer have been introduced. This allows us to situate what may be called various categories of real-world (***RW***) consumers as intermediate cases between these two extremes.

This is done by introducing the additional concepts of the sets of time-steps $\{\boldsymbol{S}_t\}$ and $\{\boldsymbol{R}_t\}$. Together $\{\boldsymbol{S}_t\}$ and $\{\boldsymbol{R}_t\}$ make up the whole time duration of interest; that is $\{\boldsymbol{S}_t\} \cup \{\boldsymbol{R}_t\} = \{\boldsymbol{T}\}$. Now $\{\boldsymbol{S}_t\}$ is defined to the set of time-steps about which the consumer is sensitive at time t while $\{\boldsymbol{R}_t\}$ is the set about which the consumer is insensitive. To put it another way, at time t the perception of benefit (consumer utility function) of this particular consumer extends over the set $\{\boldsymbol{S}_t\}$ and excludes the set $\{\boldsymbol{R}_t\}$. Fig. 1 illustrates the case of $\{\boldsymbol{S}_t\}$ extending over the nearest four time-steps (in addition to the current time-step, of course) but excluding all the more distant time-steps.

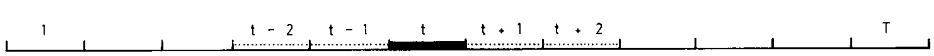

Fig. 1 *Consumer price sensitivity range*

Current time step — Members of $\{S_t\}$ — Members of $\{R_t\}$

3.2 Relationship of $\{S_t\}$ to E and F matrices

Obviously, if the perception of a consumer extends over a time range $\{S_t\}$, the elasticity matrix which measures consumer price sensitivity will have non-zero entries only within this range. The extreme ***SR*** consumer will, therefore, need to be modelled by a diagonal ***E***-matrix while the ***E***-matrix of the ***LR*** consumer may have non-zero elements anywhere. The elasticity matrix of a ***RW*** consumer will have the structure illustrated in Fig. 2 where non-zero entries are found only on and within a range $\{S_t\}$ about the diagonal on row t.

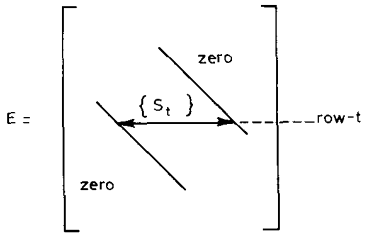

Fig. 2 *E-matrix structure*

It is necessary to appreciate that the matrix $F(F = E^{-1})$ will, however, in general have non-zero entries throughout. The numerically most significant values, however, will occur within and close to the non-zero band of the corresponding ***E***-matrix.

3.3 Summary of mathematical results

Using the concepts of E, F, η, $\{S_t\}$ and $\{R_t\}$ developed so far, and using the symbols p, D and B to represent price, quantity of electricity consumed and consumer utility derived from electricity consumption, respectively, the following results, derived in the Appendix, can be shown to hold for different categories of consumers.

SR *consumers:*

$$\frac{\partial B_t}{\partial D_t} = p_t \tag{6a}$$

$$\frac{\partial B_t}{\partial D_{t'}} = 0 \quad t \neq t' \tag{6b}$$

$$\frac{\partial B_t}{\partial p_t} = p_t e_{tt} \tag{7a}$$

$$\frac{\partial B_t}{\partial p_{t'}} = 0 \quad t \neq t' \tag{7b}$$

LR *consumers:*

$$\frac{\partial}{\partial D_t}\left(\sum_{\tau \in \{T\}} B_\tau\right) = p_t \tag{8a}$$

$$\frac{\partial}{\partial p_t}\left(\sum_{\tau \in \{T\}} B_\tau\right) = \sum_{\tau \in \{T\}} p_\tau e_{\tau t} \tag{8b}$$

RW *consumers:*

$$\frac{\partial}{\partial D_t}\left(\sum_{\tau \in \{S_t\}} B_\tau\right) = p_t \tag{9a}$$

$$\frac{\partial}{\partial D_t}\left(\sum_{\tau \in \{R_t\}} B_\tau\right) = \sum_{\tau \in \{R_t\}} \eta D_\tau f_{\tau t} \tag{9b}$$

$$\frac{\partial}{\partial p_t}\left(\sum_{\tau \in \{R_t\}} B_\tau\right) = 0 \tag{10a}$$

$$\frac{\partial}{\partial p_t}\left(\sum_{\tau \in \{S_t\}} B_\tau\right) = \sum_{\tau \in \{S_t\}} e_{\tau t} p_\tau + \sum_{\tau \in \{R_t\}} \eta D_\tau f_{t\tau} e_{\tau t} + \sum_{\substack{\tau \in \{S_t\} \\ \tau \neq t}} \eta D_\tau \left(\frac{e_{\tau t}}{e_{\tau\tau}} + f_{t\tau} e_{\tau t} - f_{\tau t} e_{tt}\right) \tag{10b}$$

Note that in eqns. 6*a* and *b*, 8*a* and 9*a* and *b*, the set of consumer demands have been treated as independent variables (and therefore prices as dependent variables). In eqns. 7*a* and *b*, 8*b* and 10*a* and *b*, the reverse has been done. Mathematically either approach is valid.

4 Example system

A simple system consisting of three generators and four consumers is used as an example. The system is shown in Fig. 3, and Table 1 gives transmission line data. Table 2 gives the cost and supply data of the three generation sources. It will be observed that a second-order polynomial has been used to model each cost function.

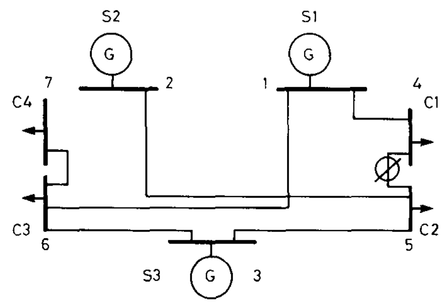

Fig. 3 *Example system*

The initial state of the system is obtained by assuming that for each consumer, demand is given for the entire duration of interest. This duration is assumed to consist of eight time-steps. The entire initial demand profile is presented in the last four columns of Table 3. Now if these demands are taken to be given, it follows that consumers are prepared to pay the RTP corresponding to these demands.

Table 1: Branch data

Bus	*R* (p.u.)	*X* (p.u.)	*B* (p.u.)	Rating (MVA)
5–4	0.000	0.013	0.000	300.0
7–6	0.065	0.050	0.003	1000.0
1–4	0.007	0.015	0.003	1000.0
2–5	0.008	0.037	0.003	1000.0
1–6	0.028	0.064	0.003	1000.0
3–5	0.008	0.031	0.003	1000.0
3–6	0.008	0.030	0.003	1000.0

Base MVA = 100

Table 2: Supplier data

Generator	P^{max} (MW)	Q^{max} (MVAR)	Cost coefficient		
			a	*b*	*c*
1	500.0	375.0	0.005	0.5	0.0
2	200.0	150.0	0.005	0.5	0.0
3	200.0	150.0	0.005	0.5	0.0

Generating cost: $f_i = a_i P_i^2 + b_i P_i + c_i$ (\$/MWhr)

Table 3: Initial power (MW)

Time-step	Supplier 1	Supplier 2	Supplier 3	Consumer 1	Consumer 2	Consumer 3	Consumer 4
1	47.47	47.17	47.76	36.00	45.00	30.00	30.00
2	79.70	79.02	80.36	60.00	75.00	50.00	50.00
3	133.80	132.10	135.50	100.00	130.00	80.00	80.00
4	119.90	118.50	121.30	90.00	110.00	75.00	75.00
5	148.30	146.20	150.40	110.00	140.00	90.00	90.00
6	125.00	123.50	126.40	95.00	120.00	75.00	75.00
7	112.40	111.20	113.60	84.00	105.00	70.00	70.00
8	79.70	79.02	80.35	60.00	75.00	50.00	50.00

The initial RTP values are obtained by executing an optimal power flow (OPF) solution for each of the time-steps. The prices imposed on consumers are given by:

consumer price = λ-value at load point
+ premium charges if capacity or
voltage limits are encountered

The second term in the above expression can be computed from the values of the Kuhn-Tucker dual variables corresponding to active inequality constraints. These values are automatically obtained as part of the OPF solution at convergence. The initial equilibrium prices found in this manner are given in Table 4 while the optimal generation values are shown in Table 3. The way in which an extended OPF model is used is described in the Appendix.

Table 4: Initial prices (\$/MWh)

Time-step	Consumer 1	Consumer 2	Consumer 3	Consumer 4
1	0.97	0.97	0.98	1.01
2	1.30	1.30	1.31	1.40
3	1.85	1.85	1.88	2.11
4	1.71	1.71	1.74	1.93
5	2.00	2.00	2.04	2.34
6	1.76	1.76	1.79	1.99
7	1.63	1.63	1.66	1.82
8	1.30	1.30	1.31	1.40

The type of tariff considered in this paper is day-ahead pricing, sometimes referred to as 24-hour update. In this approach the tariff for the next day at regular, say hourly, intervals is communicated to consumers sometime during the previous day. For convenience just eight rather than 24 time-steps have been used in the studies described here. The case of spot pricing (when no advance notice of tariff updates is given) is discussed in Reference 7.

Various changes in system operation and the response of the system to these changes will now be examined. Different types of consumer responses will be examined. The types of consumers examined will be defined by their $\boldsymbol{E}$-matrices and, where relevant, the η values and the $\{S_t\}$ and $\{\boldsymbol{R}_t\}$ sets. These are defined next.

***LR** consumer:* Defined by the following $\boldsymbol{E}$-matrix

$$\begin{bmatrix} -0.5 & 0.2 & 0.05 & 0.05 & 0.02 & 0.02 & 0.01 & 0.01 \\ 0.05 & -0.5 & 0.2 & 0.05 & 0.05 & 0.02 & 0.02 & 0.01 \\ 0.02 & 0.05 & -0.5 & 0.2 & 0.05 & 0.05 & 0.02 & 0.02 \\ 0.02 & 0.02 & 0.05 & -0.5 & 0.2 & 0.05 & 0.05 & 0.02 \\ 0.01 & 0.02 & 0.02 & 0.05 & -0.5 & 0.2 & 0.05 & 0.05 \\ 0.01 & 0.01 & 0.02 & 0.02 & 0.05 & -0.5 & 0.2 & 0.05 \\ 0.01 & 0.01 & 0.01 & 0.02 & 0.02 & 0.05 & -0.5 & 0.2 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.02 & 0.02 & 0.05 & -0.5 \end{bmatrix}$$

***SR** consumer:* Defined by a diagonal $\boldsymbol{E}$-matrix having -0.5 in the diagonal and zero everywhere else.

RW** consumers:* Three cases of ***RW consumers are used in the following studies. Known as ***RW-I***, ***RW-II*** and ***RW–III***, they are defined by the $\boldsymbol{E}$-matrices shown opposite.

The set $\{S_t\}$ for each ***RW*** case consists of the non-zero range in the $\boldsymbol{E}$-matrix. Thus ***RW-I*** is the consumer whose perception depends on current and anticipated future prices only (two steps into the future in this example). Conversely ***RW-II*** is the consumer whose behaviour is influenced by current and past events (prices) only—once again two steps into the past. ***RW-III*** is the consumer whose current consumption is affected by both past, present and future but with a horizon more limited than the ***LR*** consumer.

A constant value of $\eta = 0.2$ has been used in the examples and the $\boldsymbol{E}$-matrix of all four consumers has been assumed to be the same for any given study.

5 Response to changes

5.1 Sustained price rise

To illustrate the effects of a sustained increase in prices at one generator, the case when, right from the beginning (that is, from step 1 to step 8) instead of the expected cost function, the cost function of generator 2 was taken to be given by $a = 0.0075$, $b = 0.75$, and $c = 0$ is examined. That is, the relevant entries of the second row of Table 2 are replaced by these values.

For illustrative purposes, consumer 4 results are selected and tabulated in Table 5 and 6. The change in power

Table 5: Power comparison between *LR*, *SR* and *RW*: consumer 4 (MW)

Time-step	Initial power	*LR*	*RW*			*SR*
			I	*II*	*III*	
1	30.0	29.36	27.80	28.59	29.53	25.53
2	50.0	47.50	49.43	45.89	48.98	44.22
3	80.0	78.36	73.81	76.82	80.00	72.14
4	75.0	71.25	73.70	69.47	73.25	67.68
5	90.0	87.90	83.34	87.90	90.09	79.60
6	75.0	71.96	70.86	69.46	73.59	67.52
7	70.0	67.18	67.97	66.96	68.34	62.95
8	50.0	48.69	43.40	47.30	49.67	44.22

RW-I

$$\begin{bmatrix} -0.5 & 0.2 & 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0.2 & 0.05 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.2 & 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 & 0.2 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0.2 & 0.05 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 0.2 & 0.05 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix}$$

RW-II

$$\begin{bmatrix} -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0.2 & -0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & 0.2 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.05 & 0.2 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05 & 0.2 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.05 & 0.2 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.05 & 0.2 & -0.5 \end{bmatrix}$$

RW-III

$$\begin{bmatrix} -0.5 & 0.2 & 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & -0.5 & 0.2 & 0.05 & 0 & 0 & 0 & 0 \\ 0.05 & 0.2 & -0.5 & 0.2 & 0.05 & 0 & 0 & 0 \\ 0 & 0.05 & 0.2 & -0.5 & 0.2 & 0.05 & 0 & 0 \\ 0 & 0 & 0.05 & 0.2 & -0.5 & 0.2 & 0.05 & 0 \\ 0 & 0 & 0 & 0.05 & 0.2 & -0.5 & 0.2 & 0.05 \\ 0 & 0 & 0 & 0 & 0.05 & 0.2 & -0.5 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0.05 & 0.02 & -0.5 \end{bmatrix}$$

demand is graphed in Fig. 4. Only the ***LR***, ***SR*** and ***RW-I*** cases are included in the graph. The significant differences for different types of consumer behavioral representations is to be noted.

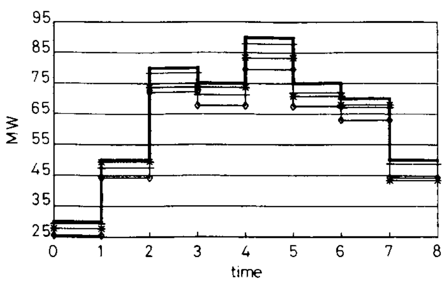

Fig. 4 *Response to sustained price change (consumer 4)*

—— Initial power
—+— LR
—*— RW-I
—◇— SW

Table 6: Price comparison between *LR*, *SR* and *RW*: consumer 4 ($/MWHr)

Time-step	Base case	*LR*	*RW* *I*	*RW* *II*	*RW* *III*	*SR*
1	1.01	1.13	1.10	1.20	1.88	1.06
2	1.40	1.53	1.55	1.58	2.48	1.45
3	2.11	2.32	2.22	2.35	3.10	2.16
4	1.93	2.09	2.14	2.10	2.68	1.98
5	2.34	2.56	2.49	2.57	3.09	2.35
6	1.99	2.16	2.19	2.11	2.86	2.04
7	1.82	1.98	2.07	1.98	2.81	1.88
8	1.40	1.54	1.54	1.52	2.29	1.45

5.2 Mid-term price rise and capacity limit

In this example the change in the supply side is imposed on the system for the duration of time-steps 5 through 8. The changes imposed are given in Table 7 from which it can be seen that, for this duration, the cost at generator 2 is assumed to have risen by 50% and the capacity of generator 3 is assumed to have decreased by 25%.

Table 7: Supplier data for steps 5 to 8

Generator	P^{max} (MW)	Q^{max} (MVAR)	Cost coefficient *a*	Cost coefficient *b*	Cost coefficient *c*
1	500.0	375.0	0.005	0.5	0.0
2	200.0	150.0	0.0075	0.75	0.0
3	150.0	112.0	0.005	0.5	0.0

Generating cost: $f_i = a_i P_i^2 + b_i P_i + c_i$ ($/MWHr)

Power and price comparison for consumer 4 are given in Tables 8 and 9 and power is graphed in Fig. 5. It is

Table 8: Power comparison between *LR*, *SR* and *RW*: consumer 4 (MW)

Time-step	Initial power	*LR*	*RW* *I*	*RW* *II*	*RW* *III*	*SR*
1	30.0	32.00	30.17	30.33	32.97	30.00
2	50.0	51.32	50.65	50.54	54.86	50.00
3	80.0	81.72	82.20	80.62	85.69	80.00
4	75.0	69.23	81.69	75.40	76.97	75.00
5	90.0	91.29	75.01	82.49	82.93	70.80
6	75.0	69.66	66.91	63.73	65.07	63.32
7	70.0	65.57	61.21	58.88	62.66	59.13
8	50.0	46.59	41.48	40.78	44.05	41.39

significant to note that, unlike in the example of Section 5.1, now increase or decrease of consumption by different types of consumer do not always occur at the same time-steps.

Table 9: Price comparison between *LR*, *SR* and *RW*: consumer 4 ($/MWHr)

Time-step	Base case	*LR*	*RW*			*SR*
			I	*II*	*II*	
1	1.01	1.05	1.02	1.11	1.75	1.01
2	1.40	1.46	1.41	1.50	2.35	1.40
3	2.11	2.20	2.16	2.19	2.89	2.11
4	1.93	2.11	2.09	1.98	2.56	1.93
5	2.34	2.31	2.31	2.33	2.89	2.62
6	1.99	2.10	2.12	1.95	2.76	2.14
7	1.82	1.90	1.95	1.80	2.72	1.97
8	1.40	1.44	1.50	1.40	2.20	1.51

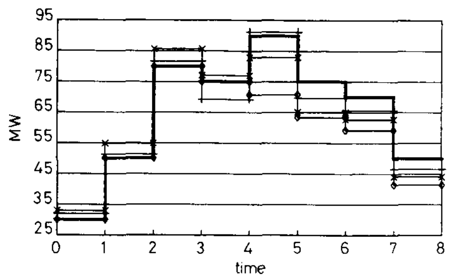

Fig. 5 *Response to mid-term price rise and capacity limit (consumer 4)*
—— Initial power
—+— LR
—×— RW-III
—◇— SW

5.3 Short-term price rise

We now consider the third example in which a cost variation occurs for one time-step only — in this case step 5. It is assumed, that for this time-step only, the cost coefficients of all generators are increased to 150% of the original value, that is $a = 0.0075$, $b = 0.75$, and $c = 0$ for all generators (see Table 2).

The changes in consumption are shown in Table 10. It is noted that for the ***SR*** case a change occurs only at time-step 5.

Table 10: Power comparison between *LR*, *SR* and *RW*: consumer 4 (MW)

Time-step	Initial power	*LR*	*RW*			*SR*
			I	*II*	*III*	
1	30.0	32.22	30.35	30.23	30.95	30.00
2	50.0	51.15	51.37	50.32	52.31	50.00
3	80.0	80.81	84.78	79.76	85.15	80.00
4	75.0	63.02	87.32	74.90	91.65	75.00
5	90.0	87.96	41.84	41.84	40.99	88.81
6	75.0	73.20	74.91	87.30	91.63	75.00
7	70.0	70.08	69.77	75.02	75.29	70.00
8	50.0	49.62	50.38	51.29	52.19	50.00

5.4 Common tariff for all consumers

In this section, the case when the consumers are released from individual tariffs and a common tariff, derived as described in the Appendix, is applied to all consumers is considered. As each consumer now faces a price slightly different from the original set in Table 4, the consumption values will alter from the initial expected values given in Table 3. The changes depend on elasticity and can be derived by using the extended OPF as described in the Appendix. The common tariff for the initial system, for the ***LR*** case only, are given in Table 11 where the modified consumption values for consumers 1 and 4 only are also shown.

6 Conclusions

This paper has been concerned with emphasising the importance of consumer representation on the overall system behaviour of real-time pricing strategies. To achieve this in a systematic way it was necessary to establish theoretically consistent mathematical models. The system response model, furthermore, required the use of an iterative OPF and an elasticity model. The application of the theory has been illustrated with several different simulation examples.

Significant insight into system behaviour with RTP-type tariffs has been gained. System response depends not only on the magnitude of consumer price response, this would be obvious, but on the time horizon of consumer rationality. Prediction of power system behaviour and proper implementation of optimal system dispatch requires new techniques such as modelling of time-dependent consumer elasticity and interaction of optimal dispatch solutions at different times.

8 References

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Table 11: Common tariff

Time-step	Initial price ($/MWHr)		Common price ($/MWHr)	Modified load (MW)	
	Consumer 1	Consumer 4		Consumer 1	Consumer 4
1	0.97	1.01	0.98	37.05	27.32
2	1.30	1.40	1.32	61.17	46.98
3	1.85	2.11	1.93	98.68	83.37
4	1.71	1.93	1.77	89.89	75.25
5	2.00	2.34	2.11	107.44	96.47
6	1.76	1.99	1.83	93.82	78.00
7	1.63	1.82	1.69	82.62	73.49
8	1.30	1.40	1.33	59.36	51.59

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9 Appendix

9.1 Marginal utility expressions

Real-time pricing requires the following extension of the conventional supply-demand economic model. Let $\boldsymbol{p} = [p_1, \ldots, p_T]^T$ be a price vector and $\boldsymbol{D} = [D_1, \ldots, D_T]^T$ a typical consumer consumption vector, with $t = 1, \ldots, T$ being the time-steps. Either the $\boldsymbol{p}$ or the $\boldsymbol{D}$ vector may be chosen as the independent variable set and the other as dependent, without violating mathematical consistency. Let B_t stand for the economist's concept of a consumer utility function (at time t).

Now B_t is defined as the area under the price-demand curve as is usually done (Fig. 6) but since a multivariable representation is involved the following generalisation is needed.

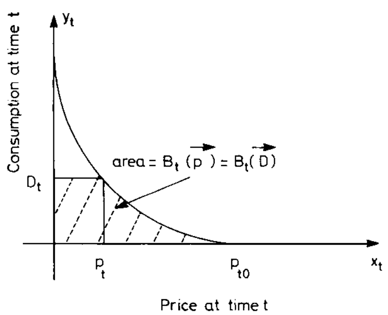

Fig. 6 *Variation of consumption at time t with price at time t, when all other prices (p_τ; $\tau \neq t$) are held constant; $y_t = y_t(p_1, \ldots, x_t, \ldots, p_T)$*

When consumption is chosen as the independent variable set, let $x_t = x_t\ (D_1, \ldots, y_t, \ldots, D_T)$ stand for the variation of price with y_t the consumption at time t, the consumption at other times remaining constant. Conversely if price is chosen as the independent variable set, let $y_t = y_t(p_1, \ldots, x_t, \ldots, p_T)$ stand for the variation of consumption with x_t, the price at time t, the price at other times remaining constant.

Now, two alternative but mathematically equivalent definitions of consumer utility can be written.

With consumption as the independent variable set

$$B_t(\boldsymbol{D}) = \int_0^{D_t} x_t\, dy_t \tag{11}$$

With price as the independent variable set,

$$B_t(\boldsymbol{p}) = p_t D_t + \int_{p_t}^{p_{t0}} y_t\, dx_t \tag{12}$$

The following partial derivations follow from these expressions.

$$\frac{\partial B_t(\boldsymbol{D})}{\partial D_t} = p_t \tag{13}$$

$$\frac{\partial B_t(\boldsymbol{D})}{\partial D_{t'}} = \int_0^{D_t} \frac{\partial x_t}{\partial D_{t'}}\, dy_t \quad t \neq t' \tag{14}$$

$$\frac{\partial B_t(\boldsymbol{p})}{\partial p_t} = p_t e_{tt} \tag{15}$$

$$\frac{\partial B_t(\boldsymbol{p})}{\partial p_{t'}} = p_t e_{tt'} + \int_{p_t}^{p_{t0}} \frac{\partial y_t}{\partial p_{t'}}\, dx_t \quad t \neq t' \tag{16}$$

where e_{tt} and $e_{tt'}$ are assumed constant and stand for elasticity of demand and cross-time elasticity of demand respectively, that is

$$e_{tt} = \frac{\partial D_t(\boldsymbol{p})}{\partial p_t} \tag{17}$$

$$e_{tt'} = \frac{\partial D_t(\boldsymbol{p})}{\partial p_{t'}} \quad t \neq t' \tag{18}$$

The following consumer behavioral descriptions can now be advanced.

(*a*) The spot rational (***SR***) consumer who is insensitive to all but current events is described by

$$\frac{\partial B_t(\boldsymbol{D})}{\partial D_t} = p_t \tag{19}$$

(marginal current utility = current price)

$$\frac{\partial B_t(\boldsymbol{p})}{\partial p_t} = p_t e_{tt} \tag{20}$$

(mathematically equivalent to eqn. 19)

$$e_{tt'} = 0 \quad \text{if } t \neq t' \tag{21}$$

(*b*) The long term rational (***LR***) or perfect consumer, not based on the B_t model derived above, is described by

$$\frac{\partial}{\partial D_t}\left[\sum_{\tau \in \{T\}} B_\tau(\boldsymbol{D})\right] = p_t \tag{22}$$

(marginal total utility with respect to current demand = current price) and by the chain rule of differentation and definition (eqn. 18) it also follows that

$$\frac{\partial}{\partial p_t}\left[\sum_{\tau \in \{T\}} B_\tau(\boldsymbol{p})\right] = \sum_{\tau \in \{T\}} p_\tau e_{t\tau} \tag{23}$$

Furthermore, any $e_{tt'}$ may be non-zero.

(*c*) A real-world (***RW***) consumer, heuristically derived from the above mathematical models and intermediate between (*a*) and (*b*). Using the set $\{\boldsymbol{S}_t\}$ as defined in Section 3.1 and Fig. 2 and setting price to match perceived marginal utility of consumption gives

$$\frac{\partial}{\partial D_t}\left[\sum_{\tau \in \{S_t\}} B_\tau(\boldsymbol{D})\right] = p_t \tag{24}$$

Furthermore, for this class of consumer, behavioral consistency demands that $e_{tt'}$ must be zero if t' is outside $\{\boldsymbol{S}_t\}$ (a consumer cannot be sensitive to benefits which are not perceived).

If, furthermore, we write

$$f_{tt} = \frac{\partial p_t(\boldsymbol{D})}{\partial D_t} \tag{25}$$

$$f_{tt'} = \frac{\partial p_t(\boldsymbol{D})}{\partial D_{t'}} \quad t \neq t' \tag{26}$$

and if we assume that e_{tt} and f_{tt} are constant while $e_{tt'}$ and $f_{tt'}$ $(t \neq t')$ remain constant for a consumption range $\pm\eta D_t$ $(0 \leqslant \eta \leqslant 1)$ about the normal consumption level D_t and zero outside this range, it can be shown from eqns. 14 and 16 and definition 17 and 18 and eqn. 26 that

$$\frac{\partial B_t(\boldsymbol{D})}{\partial D_{t'}} = \eta D_t f_{tt'} \quad \forall t \neq t' \tag{27}$$

$$\frac{\partial B_t(\boldsymbol{p})}{\partial p_{t'}} = \left(p_t + \frac{\eta D_t}{e_{tt}}\right) e_{tt'} \quad \forall t \neq t' \tag{28}$$

It then follows that

$$\frac{\partial}{\partial D_t}\left[\sum_{\tau\in\{R_t\}} B_\tau(\boldsymbol{D})\right] = \sum_{\tau\in\{R_t\}} \eta D_\tau f_{\tau t} \tag{29}$$

$$\frac{\partial}{\partial p_t}\left[\sum_{\tau\in\{R_t\}} B_\tau(\boldsymbol{p})\right] = 0 \tag{30}$$

and after some simplification it can also be shown that

$$\frac{\partial}{\partial p_t}\left[\sum_{\tau\in\{S_t\}} B_\tau(\boldsymbol{p})\right] = \sum_{\tau\in\{S_t\}} e_{\tau t} p_\tau + \sum_{\tau\in\{R_t\}} \eta D_\tau f_{t\tau} e_{\tau t} + \sum_{\substack{\tau\in\{S_t\}\\ \tau\neq t}} \eta D_\tau \left(\frac{e_{\tau t}}{e_{\tau\tau}} + f_{\tau\tau} e_{\tau t} - f_{\tau t} e_{tt}\right) \tag{31}$$

9.2 Extended OPF algorithm

An OPF algorithm is used to obtain the optimal dispatches for any given consumer demand profile. The solution also provides the values of the system-λ values and the Kuhn-Tucker dual variables (μ, π below) which allow the computation of the prices at all bus-bars at all time-steps. As a complete $T \times T$ $\boldsymbol{E}$-matrix has been assumed for each consumer elasticity representation, the consumption of each consumer can be adjusted by using an expression which is basically equivalent to

$$\Delta \boldsymbol{D} = \boldsymbol{E}(\boldsymbol{p} - \boldsymbol{p}_0) \tag{32}$$

where $\Delta \boldsymbol{D}$ is the consumption change, $\boldsymbol{p}$ the updated price vector and $\boldsymbol{p}_0$ an initial price vector. The extended OPF has now to be performed again and the above sequence of calculations iterated until convergence is obtained.

The extended OPF consists of the following mathematical programming (optimisation) problem

$$\underset{D_{jt},\, G_{it}\ \forall i, j, \tau}{\text{Maximise}} \sum_{j\in\{J\}} \sum_{\tau\in\{T\}} B_{j\tau}(\boldsymbol{D}_j) - \sum_{i\in\{I\}} \sum_{\tau\in\{T\}} C_{i\tau}(G_{i\tau}) \tag{33}$$

subject to

$$\boldsymbol{g}_t(\boldsymbol{D}_t, \boldsymbol{G}_t) = 0 \quad t = 1, \ldots, T \tag{34}$$

$$\boldsymbol{h}_t(\boldsymbol{D}_t, \boldsymbol{G}_t) \leqslant 0 \quad t = 1, \ldots, T \tag{35}$$

Here $\{\boldsymbol{J}\}$ is the set of consumers and $\{\boldsymbol{I}\}$ the set of generators, $\boldsymbol{g}_t$ the appropriate selection from the load flow equations and $\boldsymbol{h}_t$ generator and transmission capacity and bus-bar voltage limits — both sets specified for each t. Also $\boldsymbol{D}_j = [D_{j1}, \ldots, D_{jT}]$, $\boldsymbol{D}_t = [D_{1t}, \ldots, D_{Jt}]$, $\boldsymbol{G}_t = [G_{1t}, \ldots, G_{It}]$.

The following solution structure results for each class of consumers.

SR consumers, by use of eqn. 21

$$p_{jt} = \frac{\partial \boldsymbol{g}_t}{\partial D_{tj}} \boldsymbol{\Lambda}_t + \frac{\partial \boldsymbol{h}_t}{\partial D_{tj}} \boldsymbol{\Pi}_t \quad \forall j, t \tag{36}$$

For ***LR*** consumers, by use of eqn. 22, the same expression results — but *not* the same overall problem as explained below.

RW consumers, by use of eqns. 27 and 29

$$p_{jt} = \left[\sum_{\tau\in\{R_t\}} -\eta D_{j\tau} f_{j\tau t}\right] + \frac{\partial \boldsymbol{g}_t}{\partial D_{tj}} \boldsymbol{\Lambda}_t + \frac{\partial \boldsymbol{h}_t}{\partial D_{tj}} \boldsymbol{\pi}_t \quad \forall j, t \tag{37}$$

In eqns. 36 and 37, $\boldsymbol{\Lambda}_t$, $\boldsymbol{\Pi}_t$ are the dual variable vectors at time t corresponding to eqns. 34 and 35 respectively.

This set of prices is now used to update the consumption of each consumer by eqn. 32 before performing another OPF iteration. In the case of the ***SR*** consumer, as the $\boldsymbol{E}$-matrix is diagonal, consumption is affected only by current price, hence eqn. 36 actually amounts to a set of independent OPF problems decoupled in time. In the ***LR*** and ***RW*** cases, there arise a set of cross-time dependent problems because of the non-diagonal structure of the $\boldsymbol{E}$-matrix — price change at any time, in general, affecting consumption at all times. In all cases, however, an iterative procedure is required.

The extended OPF objective function in eqn. 33 is unsuitable for the case, probably of great practical interest, where the same tariff is to be applied to all consumers irrespective of their location. In this case eqn. 33 is replaced by eqn. 38 below while eqns. 34 and 35 of the mathematical programming problem remain unchanged.

$$\underset{\boldsymbol{p},\, G_{it}\ \forall i, \tau}{\text{Maximise}} \sum_{j\in\{J\}} \sum_{\tau\in\{T\}} B_{j\tau}(\boldsymbol{p}) - \sum_{i\in\{I\}} \sum_{\tau\in\{T\}} C_{i\tau}(G_{i\tau}) \tag{38}$$

After some algebra and using eqns. 30 and 31, it can be shown that the elements p_τ of the common price vector $\boldsymbol{p}$ are given for ***RW*** consumers by the solutions to the simultaneous set

$$\sum_{j\in\{J\}} \sum_{\tau\in\{S_t\}} p_\tau e_{\tau t j} = \sum_{j\in\{J\}} \left\{ \sum_{\tau\in\{T\}} \left[e_{\tau t j}\left(\frac{\partial \boldsymbol{g}_\tau}{\partial D_{\tau j}} \boldsymbol{\Lambda}_\tau + \frac{\partial \boldsymbol{h}_\tau}{\partial D_{\tau j}} \boldsymbol{\pi}_\tau\right)\right] - \sum_{\tau\in\{R_t\}} \eta D_\tau f_{t\tau} e_{\tau t} \times \sum_{\substack{\tau\in\{S_t\}\\ \tau\neq t}} \left[\eta D_\tau\left(\frac{e_{\tau t}}{e_{tt}} + f_{\tau\tau} e_{\tau t} - f_{\tau t} e_{tt}\right)\right]\right\} \quad \forall t \tag{39}$$

For ***LR*** consumers, the simpler result

$$\sum_{j\in\{J\}} \sum_{\tau\in\{T\}} p_\tau e_{\tau t j} = \sum_{j\in\{J\}} \left\{ \sum_{\tau\in\{T\}} \left[e_{\tau t j}\left(\frac{\partial \boldsymbol{g}_\tau}{\partial D_{\tau j}} \boldsymbol{\Lambda}_\tau + \frac{\partial \boldsymbol{h}_\tau}{\partial D_{\tau j}} \boldsymbol{\Pi}_\tau\right)\right]\right\} \quad \forall t \tag{40}$$

can be obtained, while for ***SR*** consumers, the result, eqn. 41 below, is even simpler.

$$\sum_{j\in\{J\}} p_t e_{ttj} = \sum_{j\in\{J\}} \left[e_{ttj}\left(\frac{\partial \boldsymbol{g}_t}{\partial D_{tj}} \boldsymbol{\Lambda}_t + \frac{\partial \boldsymbol{h}_t}{\partial D_{tj}} \boldsymbol{\Pi}_t\right)\right] \quad \forall t \tag{41}$$

It is to be observed that the following expression

$$\frac{dC_{ti}}{dG_{ti}} = -\frac{\partial \boldsymbol{g}_t}{\partial G_{ti}} \boldsymbol{\Lambda}_t - \frac{\partial \boldsymbol{h}_t}{\partial G_{ti}} \Pi_t \quad \forall i, t \tag{42}$$

which describes the supply side situation, remains unchanged for all types of consumer rationality and consumer pricing models. An OPF solution consists of a simultaneous solution of the equation set 42 with one set selected from the sets of equations 36, 37 and 39–41 as relevant. Because of the iterative and intertemporal nature of the solution structure it is referred to as an extended OPF.